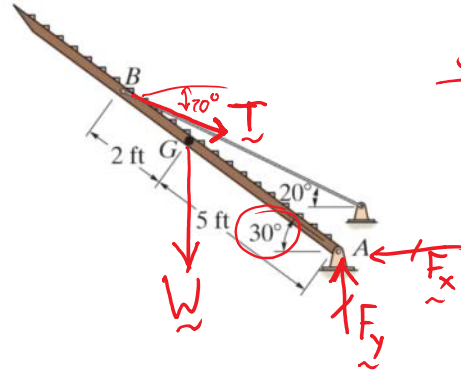


The uniform truck ramp has weight 400 lb and is pinned to the body of the truck at each side and held in the position shown by the two side cables. Determine the reaction forces at the pins and the tension in the cables.



3 unknowns
 T, F_x, F_y
 One other force
 W (weight of ramp)

Sum forces:

In x-direction

$$\sum F_x = 0 \Rightarrow T \cdot \cos(20^\circ) - F_x = 0$$

$$\Rightarrow F_x = T \cdot \cos(20^\circ)$$

In y-direction

$$\sum F_y = -W - T \cdot \sin(20^\circ) + F_y = 0$$

Sum moments about A: $\sum M_A = 0$

Zero moment created by F_x and F_y

$$W \cdot (5') \cdot \cos(30^\circ) + (T \cdot \sin(20^\circ)) \cdot (7' \cdot \cos 30^\circ) - (T \cdot \cos(20^\circ)) \cdot (7' \cdot \sin(30^\circ)) = 0$$

} 1 eqn.
} 1 unknown

algebra

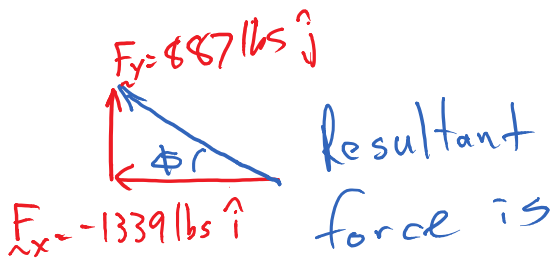
$$W \cdot 5 \cdot \cos 30^\circ$$

$$T = \frac{W \cdot 5 \cdot \cos 30^\circ}{7 \cdot (\cos 20^\circ \cdot \sin 30^\circ - \sin 20^\circ \cdot \cos 30^\circ)}$$

$$T = 1425 \text{ lbs}$$

Solve $F_x = T \cdot \cos(20^\circ) = 1339 \text{ lbs}$

$$F_y = W + T \cdot \sin 20^\circ = 887 \text{ lbs}$$

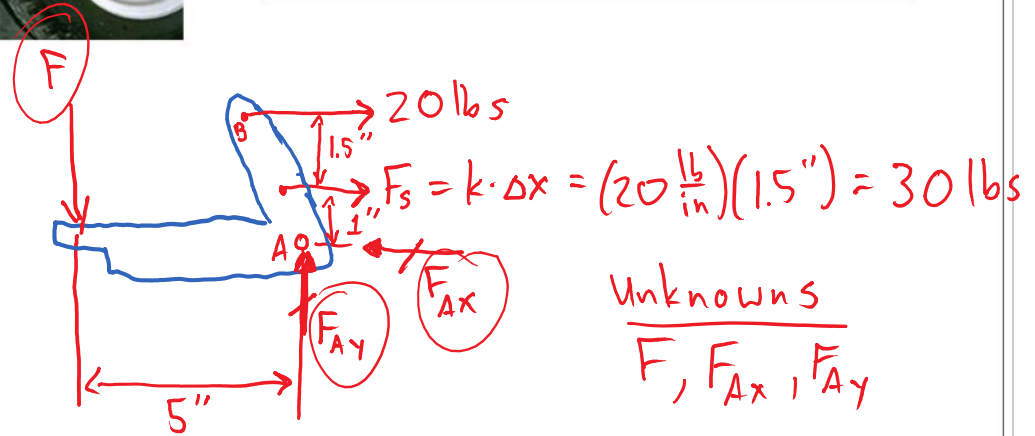
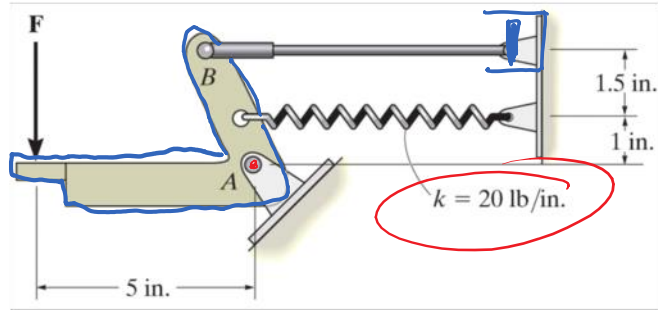


$$\vec{F} = (-1339\hat{i} + 887\hat{j}) \text{ lb}$$

Find ϕ : $\phi = \tan^{-1}\left(\frac{887}{1339}\right) = 33.5^\circ!$

NOT $30^\circ!$

The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force in the short link at B is 20 lb. Determine the vertical force applied to the pedal.



$$\sum F_x = -F_{Ax} + (20 \text{ lbs}) + (30 \text{ lbs}) = 0$$

$$\Rightarrow F_{Ax} = 50 \text{ lbs}$$

$$\sum F_y = -F + F_{Ay} = 0 \Rightarrow F_{Ay} = F$$

$$\sum M_A = (5'') \cdot F - (1'')(30 \text{ lbs}) - (2.5'')(20 \text{ lbs}) = 0$$

$$5 \cdot F = 30 \text{ lbs} + 50 \text{ lbs} = 80 \text{ lbs}$$

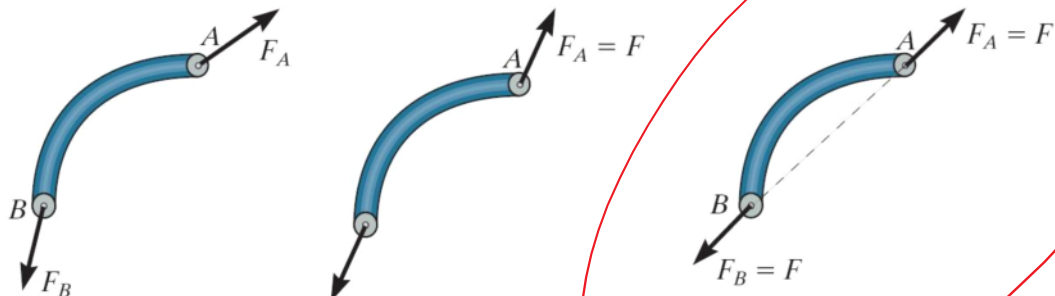
$$\Rightarrow \boxed{F = \frac{80}{5} \text{ lbs} = 16 \text{ lbs}}$$

Note: $\sum F_x$ & $\sum F_y$ was not
necessary to arrive
at the solution.

Two-force members

As the name implies, two-force members have forces applied at only two points.

If we apply the equations of equilibrium to such members, we can quickly determine that **the resultant forces at A and B must be equal in magnitude and act in the opposite directions along the line joining points A and B.**



$$\sum \vec{F} = \vec{F}_A + \vec{F}_B = 0$$

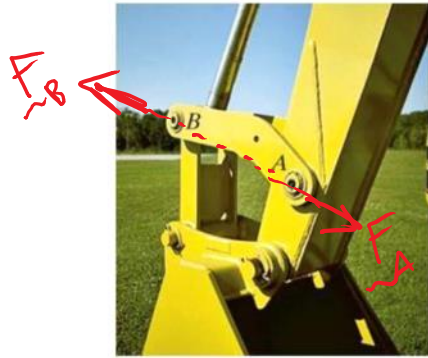
$$\Rightarrow \vec{F}_A = -\vec{F}_B$$

$= F_A (\cos\theta \hat{i} + \sin\theta \hat{j})$
 $= F_B (-\cos\theta \hat{i} - \sin\theta \hat{j})$

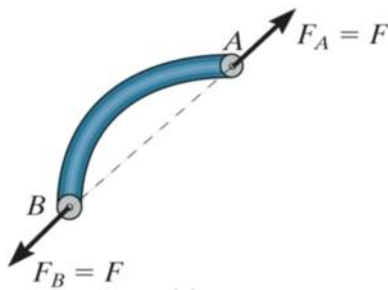
this expression tells us that \vec{F}_A & \vec{F}_B are:

1. collinear
2. equal in magnitude
3. opposite in direction
4. create no moment

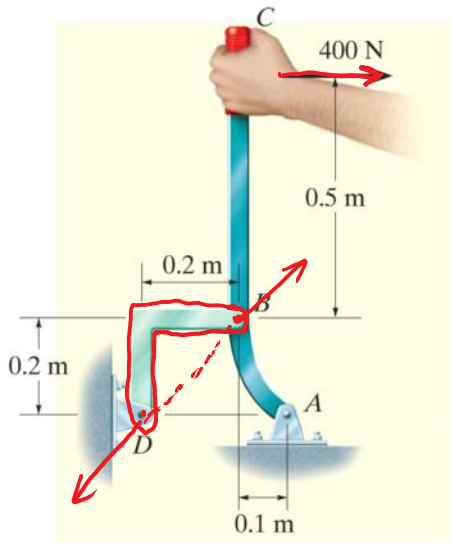
Examples of two-force members



In the cases above, members AB can be considered as two-force members, provided that their weight is neglected.



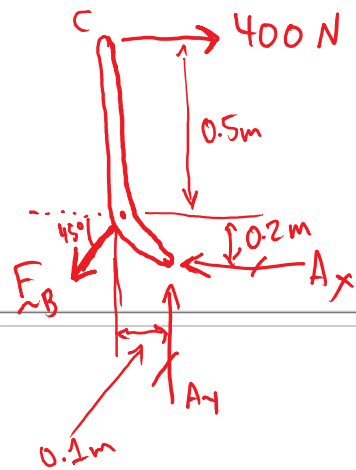
This fact **simplifies** the equilibrium analysis of some rigid bodies since the directions of the resultant forces at A and B are thus known (along the line joining points A and B).



The lever ABC is pin supported at A and connected to a short link BD. If the weight of the members is negligible, determine the reaction forces at pins D and A.

- ① Is the lever ABC a 2-force member?
 A) Yes
 B) No
- ② Is the link BD a 2-force member?
 A) Yes
 B) No

FBD of ABC



$$F_{Bx} = F_B \cdot \frac{\sqrt{2}}{2}$$

$$F_{By} = F_B \cdot \frac{\sqrt{2}}{2}$$

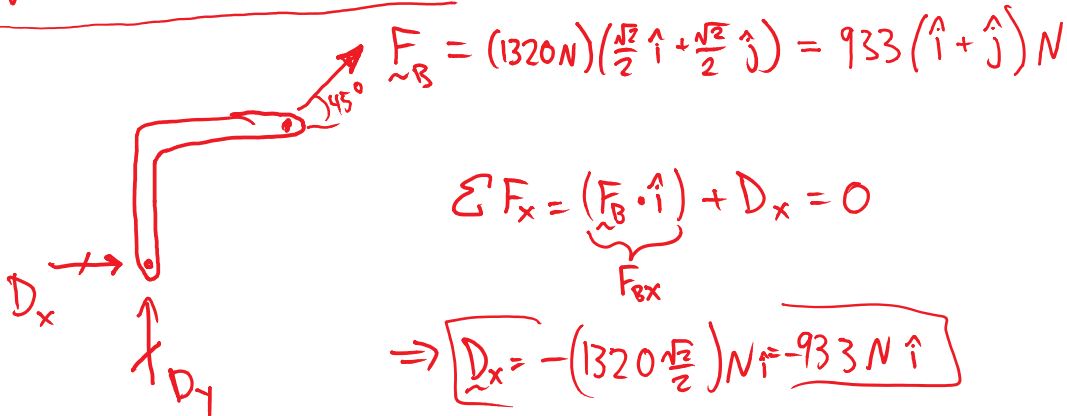
$$\sum M_A = -(0.7\text{m})(400\text{N}) + (0.2\text{m})(F_B \frac{\sqrt{2}}{2}) + (0.1\text{m})(F_B \frac{\sqrt{2}}{2}) = 0$$

moment caused by horiz. component of F_B moment caused by vertical component of F_B

$$-280\text{N} + F_B \cdot \frac{\sqrt{2}}{2} (0.2\text{m} + 0.1\text{m}) = 0$$

$$F_B = 1320\text{N}$$

Reaction at D



$$F_{\sim B} = (1320\text{N}) \left(\frac{\sqrt{2}}{2} \hat{i} + \frac{\sqrt{2}}{2} \hat{j} \right) = 933 (\hat{i} + \hat{j}) \text{N}$$

$$\sum F_x = (F_{\sim B} \cdot \hat{i}) + D_x = 0$$

F_{Bx}

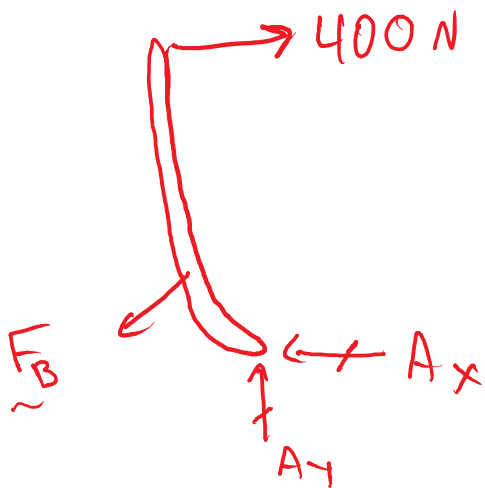
$$\Rightarrow D_x = - \left(1320 \frac{\sqrt{2}}{2} \right) \text{N} = -933\text{N}$$

$$\sum F_y = (F_B \cdot \hat{j}) + D_y = 0$$

$$\Rightarrow \boxed{D_y = -933 \text{ N } \hat{j}}$$

So the reaction force is $\underline{D} = -933(\hat{i} + \hat{j}) \text{ N}$, which equals $-\underline{F}_B$, as expected, because the short link \overline{BD} is a two-force member.

Reaction at A



$$\sum F_x = 400 \text{ N} - A_x - 1320 \frac{\sqrt{2}}{2} \text{ N} = 0$$

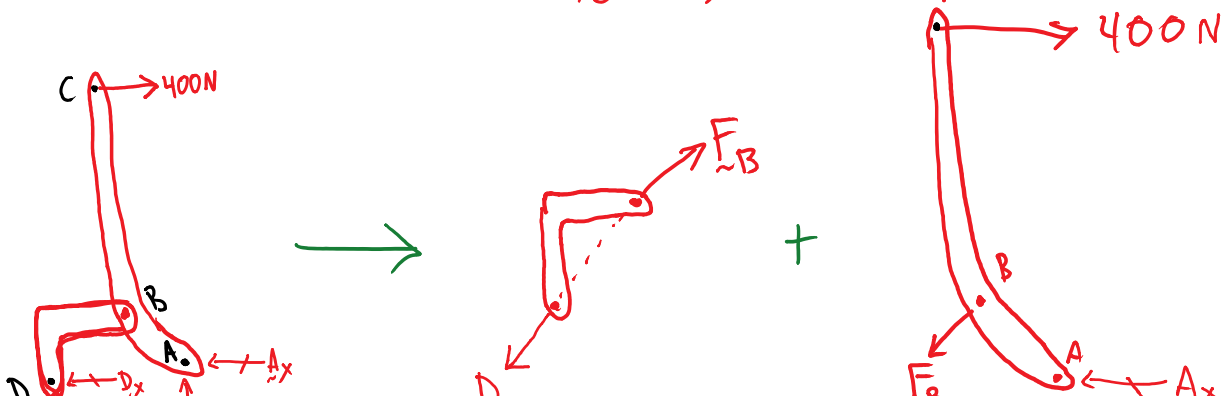
$$\Rightarrow A_x = 933 \text{ N} - 400 \text{ N}$$

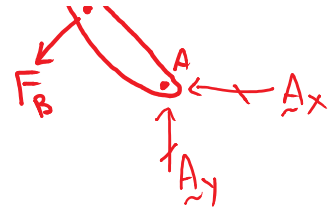
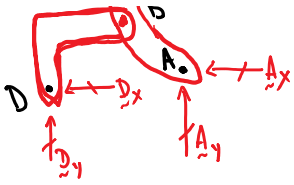
$$A_x = 533 \text{ N to the left}$$

$$\sum F_y = A_y - (1320 \frac{\sqrt{2}}{2}) \text{ N} = 0$$

$$\boxed{A_y = 933 \text{ N acting upward}}$$

Student Question: Why do we write \underline{F}_B in opposite directions acting on \overline{ABC} vs. on \overline{BD} ?





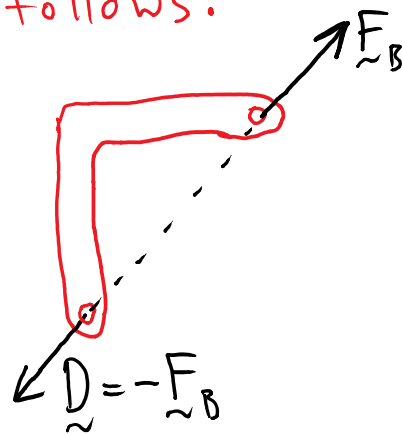
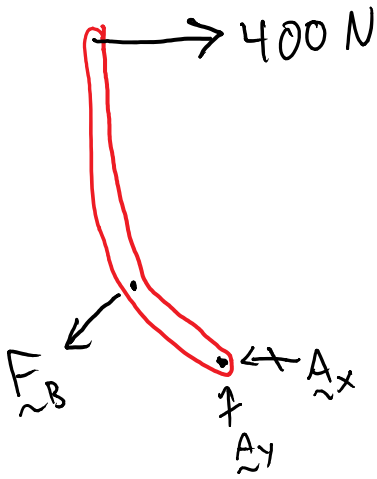
Answer 1: Newton's 3rd Law:

For every action (force), there is always an equal and opposite reaction (force).

Answer 2:

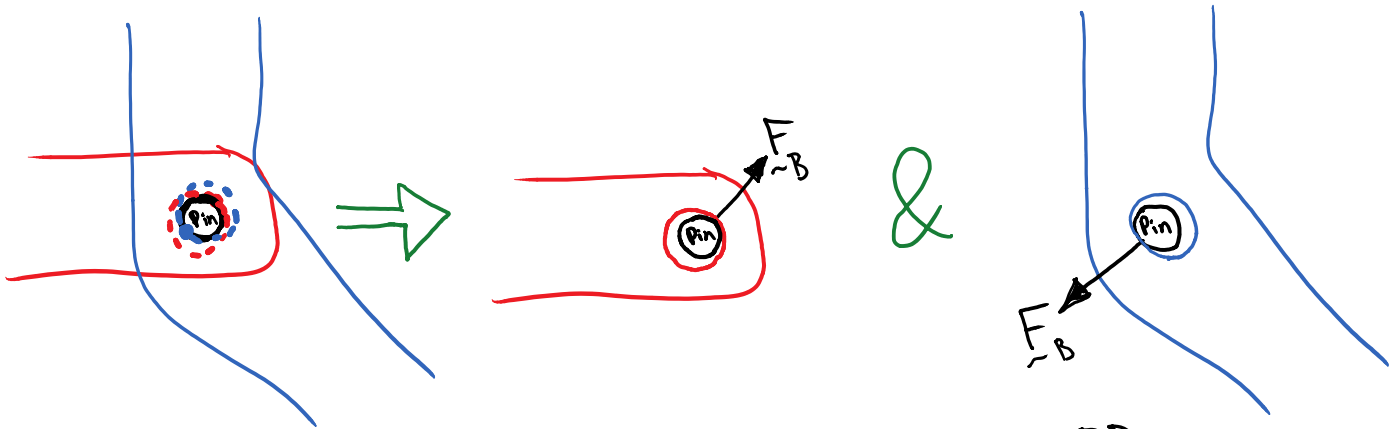
Consider firstly how the link acts against the lever. As the operator pulls, the link at B resists the motion. So the force \vec{F}_B acts in a sense that counteracts the 400 N pulling force.

Next, consider what the short link BD "feels". It is "stretched", so it must carry a tension force. Its free-body diagram must be as follows:

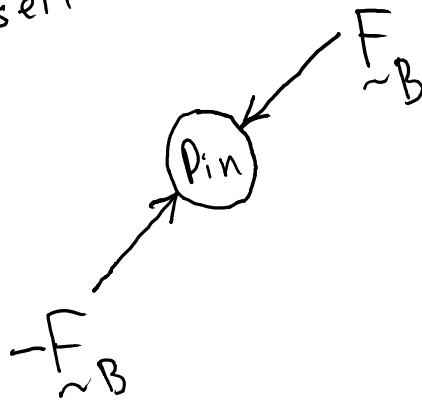


Answer 3: Consider the pin itself. Each of the members \overline{AB} and \overline{BD} have holes through which the pin

\overline{ABC} and \overline{BD} have holes through which the pin is inserted. In order to fit and allow frictionless rotation, the pin diameter must be less than the hole diameter.



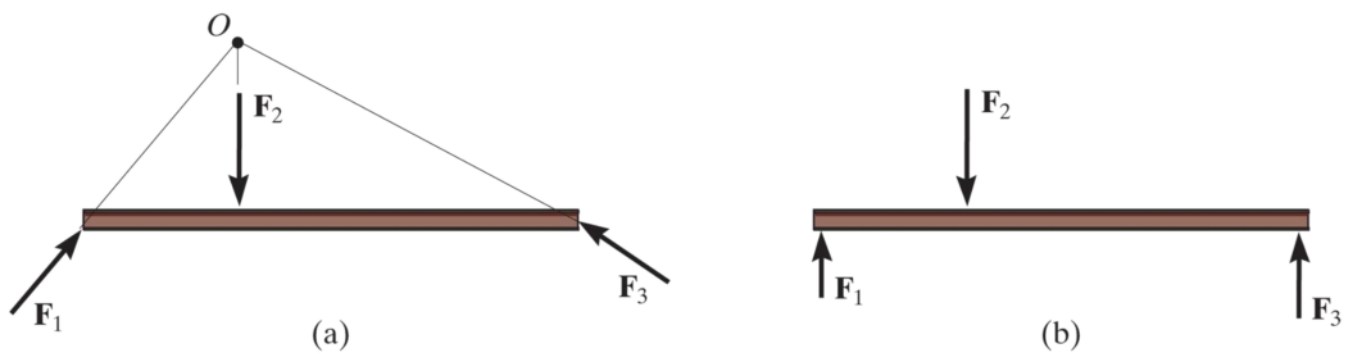
The pin carries the force from ABC to link BD.
The pin is itself a two-force member.



Three-force members

As the name implies, three-force members have forces applied at only three points.

Moment equilibrium can be satisfied only if the three forces are concurrent or parallel force system



Three-force member